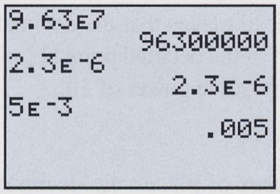
Scientific Notation

Archimedes (287–212 B.C.) was a brilliant Greek inventor and mathematician who studied at the Egyptian city of Alexandria, then the center of the scientific world. Archimedes used his knowledge of mathematics to calculate for King Gelon the number of grains of sand in the universe: 1 followed by 63 zeros. Of course, Archimedes' universe was different from our universe, and he performed his computations with Greek letter numerals, since the modern number system and scientific notation had not yet been invented. Although it is impossible to calculate the number of grains of sand in the universe, this story illustrates how long scientists have been interested in quantities ranging in size from the diameter of our galaxy to the diameter of an atom. Scientific notation offers a convenient way of expressing very large or very small numbers.



■ Figure P.16

In scientific notation, a positive number is written as a product of a number between 1 and 10 and a power of 10. For example, 9.63×10^{7} and 2.3×10^{-6} are numbers written in scientific notation.

These numbers are shown on a graphing calculator in scientific notation in Fig. P.16. Note that only the power of 10 shows in scientific notation on a calculator. The graphing calculator shown in Fig. P.16 converts to standard notation when the ENTER key is pressed (provided the number is neither too large nor too small). □

Conversion of numbers from scientific notation to standard notation is actually just multiplication.

Example 5 Scientific notation to standard notation

Convert each number to standard notation.

a. 9.63×10^{7} **b.** 2.3×10^{-6}

Solution

a. $9.63 \times 10^{7} = 9.63 \times 10{,}000{,}000$ Evaluate 10^{7}.

$= 96{,}300{,}000$ Move decimal point seven places to the right.

b. $2.3 \times 10^{-6} = 2.3 \times \frac{1}{1{,}000{,}000}$ Evaluate 10^{-6}.

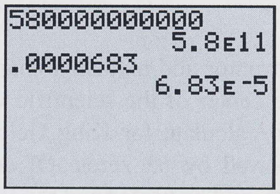
$= 2.3 \times 0.000001$

$= 0.0000023$ Move decimal point six places to the left.

Try This. Convert 3.78×10^{-2} to standard notation. ■

Observe how the decimal point is relocated in Example 5. Converting from scientific notation to standard notation is simply a matter of moving the decimal point. To convert a number from scientific to standard notation, we move the decimal point the number of places indicated by the power of 10. Move the decimal point to the right for a positive power and to the left for a negative power. Note that in scientific notation a number greater than 10 is written with a positive power of 10 and a number less than 1 is written with a negative power of 10. Numbers between 1 and 10 are not written in scientific notation.

In the next example, we convert from standard notation to scientific notation by reversing the process used in Example 5.



■ Figure P.17

A graphing calculator converts to scientific notation when you press ENTER as shown in Fig. P.17. □

Example 6 Standard notation to scientific notation

Convert each number to scientific notation.

a. 580,000,000,000 **b.** 0.0000683

Solution

a. Determine the power of 10 by counting the number of places that the decimal must move so that there is a single nonzero digit to the left of the decimal point (11 places). Since 580,000,000,000 is larger than 10, we use a positive power of 10:

$$580{,}000{,}000{,}000 = 5.8 \times 10^{11}$$

b. Determine the power of 10 by counting the number of places the decimal must move so that there is a single nonzero digit to the left of the decimal point (five places). Since 0.0000683 is smaller than 1, we use a negative power of 10:

$$0.0000683 = 6.83 \times 10^{-5}$$

Try This. Convert 5,480,000 to scientific notation. ■

One advantage of scientific notation is that the rules of exponents can be used when performing certain computations involving scientific notation. Calculators can be used to perform computations with scientific notation, but it is good to practice some computation without a calculator.

Example 7 Using scientific notation in computations

Perform the indicated operations without a calculator. Write your answers in scientific notation. Check your answers with a calculator.

a. $(4 \times 10^{13})(5 \times 10^{-9})$ **b.** $\dfrac{1.2 \times 10^{-9}}{4 \times 10^{-7}}$ **c.** $\dfrac{(2{,}000{,}000{,}000)^3(0.00009)}{600{,}000{,}000}$

Solution

a.
$$\begin{aligned}
(4 \times 10^{13})(5 \times 10^{-9}) &= 20 \times 10^{13+(-9)} && \text{Product rule for exponents}\\
&= 20 \times 10^{4} && \text{Simplify the exponent.}\\
&= 2 \times 10^{1} \times 10^{4} && \text{Write 20 in scientific notation.}\\
&= 2 \times 10^{5} && \text{Product rule for exponents}
\end{aligned}$$

b.
$$\begin{aligned}
\frac{1.2 \times 10^{-9}}{4 \times 10^{-7}} &= \frac{1.2}{4} \times 10^{-9-(-7)} && \text{Quotient rule for exponents}\\
&= 0.3 \times 10^{-2} && \text{Simplify the exponent.}\\
&= 3 \times 10^{-3} && \text{Use } 0.3 = 3 \times 10^{-1}.
\end{aligned}$$

c. First convert each number to scientific notation, then use the rules of exponents to simplify:

$$\frac{(2{,}000{,}000{,}000)^3(0.00009)}{600{,}000{,}000} = \frac{(2 \times 10^9)^3(9 \times 10^{-5})}{6 \times 10^8}$$

$$= \frac{(8 \times 10^{27})(9 \times 10^{-5})}{6 \times 10^8}$$

$$= \frac{72 \times 10^{22}}{6 \times 10^8} = 12 \times 10^{14} = 1.2 \times 10^{15}$$

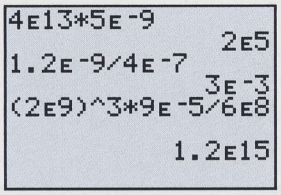


Figure P.18

These three computations are done on a calculator in Fig. P.18. Set the mode to scientific to get the answers in scientific notation.

Try This. Find the product $(7 \times 10^{14})(5 \times 10^{-3})$. ■

In the next example we use scientific notation to perform the type of computation performed by Archimedes when he attempted to determine the number of grains of sand in the universe.

Example 8 The number of grains of sand in Archimedes' earth

If the radius of the earth is approximately 6.38×10^3 kilometers and the radius of a grain of sand is approximately 1×10^{-3} meters, then what number of grains of sand have a volume equal to the volume of the earth?

Solution

Since the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, the volume of the earth is

$$\frac{4}{3}\pi(6.38 \times 10^3)^3 \text{ km}^3 \approx 1.09 \times 10^{12} \text{ km}^3.$$

Since 1 km $= 10^3$ m, the radius of a grain of sand is 1×10^{-6} km and its volume is

$$\frac{4}{3}\pi(1 \times 10^{-6})^3 \text{ km}^3 \approx 4.19 \times 10^{-18} \text{ km}^3.$$

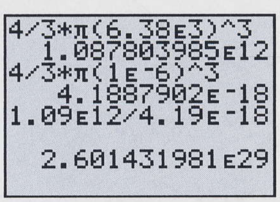


Figure P.19

To get the number of grains of sand, divide the volume of the earth by the volume of a grain of sand:

$$\frac{1.09 \times 10^{12} \text{ km}^3}{4.19 \times 10^{-18} \text{ km}^3} \approx 2.60 \times 10^{29}$$

See Fig. P.19 for the computations.

Try This. Find the volume of a sphere that has radius 2.4×10^{-3} in. ■

For Thought

True or False? Explain. Do Not Use a Calculator.

1. $2^{-1} + 2^{-1} = 1$ T

2. $2^{100} = 4^{50}$ T

3. $9^8 \cdot 9^8 = 81^8$ T

4. $(0.25)^{-1} = 4$ T

5. $\dfrac{5^{10}}{5^{-12}} = 5^{-2}$ F

6. $2 \cdot 2 \cdot 2 \cdot 2^{-1} = \dfrac{1}{16}$ F

7. $-3^{-3} = -\dfrac{1}{27}$ T

8. $\left(\dfrac{3}{4}\right)^{-2} = \left(\dfrac{4}{3}\right)^2$ T

9. $10^{-4} = 0.00001$ F

10. $98.6 \times 10^8 = 9.86 \times 10^7$ F

P.2 Exercises

Evaluate each expression without using a calculator. Check your answers with a calculator. (Example 1)

1. 3^{-1} 1/3

2. 2^{-1} 1/2

3. -4^{-2} −1/16

4. -5^{-3} −1/125

5. $\dfrac{1}{2^{-3}}$ 8

6. $\dfrac{1}{10^{-3}}$ 1000

7. $\left(\dfrac{3}{2}\right)^{-3}$ 8/27

8. $\left(\dfrac{2}{3}\right)^{-2}$ 9/4

9. $\left(-\dfrac{1}{2}\right)^{-2}$ 4

10. $\left(-\dfrac{1}{3}\right)^{-4}$ 81

11. $2^{-1} \cdot 4^2 \cdot 10^{-1}$ 4/5

12. $4 \cdot 4 \cdot 4 \cdot 4^{-1}$ 16

13. $\dfrac{3^{-2}}{6^{-3}}$ 24

14. $\dfrac{3^{-1}}{2^3}$ 1/24

15. $2^0 + 2^{-1}$ 3/2

16. $6^{-1} + 5^{-1}$ 11/30

17. $-2 \cdot 10^{-3}$ −1/500

18. $-1^{-1} \cdot (-2)^{-2}$ −1/4

Simplify each expression. (Example 2)

19. $(-3x^2y^3)(2x^9y^8)$ $-6x^{11}y^{11}$

20. $(-6a^7b^4)(3a^3b^5)$ $-18a^{10}b^9$

21. $x^2x^4 + x^3x^3$ $2x^6$

22. $a^7a^5 + a^3a^9$ $2a^{12}$

23. $(-2b^2)(3b^3) + (5b^3)(-3b^2)$ $-21b^5$

24. $(4y^6)(-xy^4) + (3xy^9)(5y)$ $11xy^{10}$

25. $(-2a^2)(a^9) - (a^5)(-4a^6)$ $2a^{11}$

26. $(w^2)(-3w) - (-5w)(-7w^2)$ $-38w^3$

27. $(-m^2)(-m) - m(-m) + m(3m^2)$ $4m^3 + m^2$

28. $(z^2)(-z) - (-z) - z(-z^2) + z(2z)$ $2z^2 + z$

29. $5^2 \cdot 3^2$ 225

30. $2^3 \cdot 4^2$ 128

Simplify each expression. Write answers without negative exponents. Assume that all variables represent nonzero real numbers. (Example 3)

31. $\left(\dfrac{1}{2}x^{-4}y^3\right)\left(\dfrac{1}{3}x^4y^{-6}\right)$ $\dfrac{1}{6y^3}$

32. $\left(\dfrac{1}{3}a^{-5}b\right)(a^4b^{-1})$ $\dfrac{1}{3a}$

33. $\dfrac{6x^7}{2x^3}$ $3x^4$

34. $\dfrac{-9x^2y}{3xy^2}$ $-\dfrac{3x}{y}$

35. $\dfrac{-3m^{-1}n}{-6m^{-1}n^{-1}}$ $\dfrac{n^2}{2}$

36. $\dfrac{-p^{-1}q^{-1}}{-3pq^{-3}}$ $\dfrac{q^2}{3p^2}$

37. $(2a^2)^3 + (-3a^3)^2$ $17a^6$

38. $(b^{-4})^2 - (-b^{-2})^4$ 0

39. $-1(2x^3)^2$ $-4x^6$

40. $(-3y^{-1})^{-1}$ $-\dfrac{y}{3}$

41. $\left(\dfrac{-2x^2}{3}\right)^3$ $-\dfrac{8x^6}{27}$

42. $\left(\dfrac{-1}{2a}\right)^{-2}$ $4a^2$

43. $\left(\dfrac{y^2}{5}\right)^{-2}$ $\dfrac{25}{y^4}$

44. $\left(-\dfrac{y^2}{2a}\right)^4$ $\dfrac{y^8}{16a^4}$

45. $\left(\dfrac{6xy^2}{8x^{-4}y^3}\right)^{-3}$ $\dfrac{64y^3}{27x^{15}}$

46. $\left(-\dfrac{15x^{-2}y^9}{18x^2y^3}\right)^{-2}$ $\dfrac{36x^8}{25y^{12}}$

Simplify each expression. Assume that all bases are nonzero real numbers and all exponents are integers. (Example 4)

47. $(x^{b-1})^3(x^{b-4})^{-2}$ x^{b+5}

48. $(a^2)^{m+2}(a^3)^{4m}$ a^{14m+4}

49. $(-5a^{2t}b^{-3t})^3$ $-\dfrac{125a^{6t}}{b^{9t}}$

50. $(-2x^{-5v}y^3)^2$ $\dfrac{4y^6}{x^{10v}}$

51. $\dfrac{-9x^{3w}y^{9v}}{6x^{8w}y^{3v}}$ $-\dfrac{3y^{6v}}{2x^{5w}}$

52. $\dfrac{6c^{9s}d^{4t}}{-9c^{3s}d^{8t}}$ $-\dfrac{2c^{6s}}{3d^{4t}}$

53. $\left(\dfrac{a^{s+2}}{a^{2s-3}}\right)^4$ a^{-4s+20}

54. $\left(\dfrac{x^{2a-3}}{x^{-4a+1}}\right)^{-4}$ $x^{-24a+16}$

Convert each number given in standard notation to scientific notation and each number given in scientific notation to standard notation. (Examples 5 and 6)

55. 4.3×10^4 43,000

56. 5.98×10^5 598,000

57. 3.56×10^{-5} 0.0000356

58. 9.333×10^{-9} 0.000000009333

59. 5,000,000 5×10^6

60. 16,587,000 1.6587×10^7

61. 0.0000672 6.72×10^{-5}

62. 0.000000981 9.81×10^{-7}

63. 7×10^{-9} 0.000000007

64. 6×10^{-3} 0.006

65. 20,000,000,000 2×10^{10}

66. 0.00000000004 4×10^{-11}

Perform the indicated operations without a calculator. Write your answers in scientific notation. Use a calculator to check. (Example 7)

67. $(5 \times 10^8)(4 \times 10^7)$ 2×10^{16}

68. $(5 \times 10^{-10})(6 \times 10^5)$ 3×10^{-4}

69. $\dfrac{8.2 \times 10^{-6}}{4.1 \times 10^{-3}}$ 2×10^{-3}

70. $\dfrac{9.3 \times 10^{12}}{3.1 \times 10^{-3}}$ 3×10^{15}

71. $5(2 \times 10^{-10})^3$ 4×10^{-29}

72. $2(2 \times 10^8)^{-4}$ 1.25×10^{-33}

73. $\dfrac{(2{,}000{,}000)^3(0.000005)}{(0.00002)^2}$ 1×10^{23}

74. $\dfrac{(6{,}000{,}000)^2(0.000003)^{-3}}{(2000)^3(1{,}000{,}000)}$ 1.7×10^{14}

Use a calculator to perform the indicated operations. Give your answers in scientific notation. (Example 8)

75. $(4.32 \times 10^{-9})(2.3 \times 10^4)$ 9.936×10^{-5}

76. $(2.33 \times 10^{23})(3.98 \times 10^{-9})$ 9.2734×10^{14}

77. $\dfrac{(5.63 \times 10^{-6})^3(3.5 \times 10^7)^{-4}}{\pi(8.9 \times 10^{-4})^2}$ 4.78×10^{-41}

78. $\dfrac{\pi(2.39 \times 10^{-12})^2}{(6.75 \times 10^{-8})^3}$ 5.83×10^{-2}

Solve each problem. Use your calculator.

79. *Body-Mass Index* The body mass index (BMI) is used to assess the relative amount of fat in a person's body (Shape Up America, www.shapeup.org). If w is weight in pounds and h is height in inches, then

$$BMI = 703wh^{-2}.$$

Find the BMI for Dallas Cowboys' player Flozell Adams at 6'7" and 335 pounds. BMI = 37.7

80. *Abnormal Fat* The accompanying table shows the height and weight of five Dallas Cowboys players in training camp (ESPN, www.espn.com). Some physicians consider your body fat abnormal if your BMI (from the previous exercise) satisfies

$$|BMI - 23| > 3.$$

Which of the five players does not have abnormal body fat? Smith

■ **Table for Exercise 80**

Player	Ht	Wt
Adams	6-7	335
Allen	6-3	326
Anderson	6-2	215
Robinson	6-4	250
Smith	5-11	175

81. *Displacement-Length Ratio* The displacement-length ratio D indicates whether a sailboat is relatively heavy or relatively light (Ted Brewer Yacht Design, www.tedbrewer.com). D is given by the formula

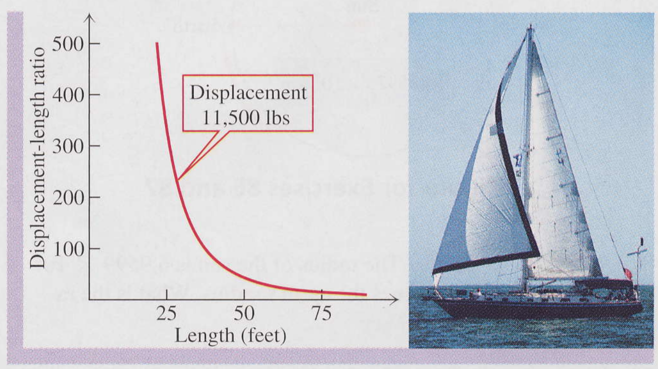
$$D = \frac{dL^{-3} \cdot 10^6}{2240},$$

where L is the length at the water line in feet, and d is the displacement in pounds. If $D > 300$, then a boat is considered relatively heavy. Find D for the USS Constitution, the nation's oldest floating ship, which has a displacement of 2200 tons and a length of 175 feet. $D = 366.5$

82. *A Lighter Boat* If $D < 150$ (from the previous exercise), then a boat is considered relatively light.

a. Use the accompanying graph to estimate the length for which $D < 150$ for a boat with a displacement of 11,500 pounds. Length greater than 30 ft

b. Use the formula to find D for a 50-foot boat with a displacement of 11,500 pounds. 41.1



■ **Figure for Exercise 82**

83. *National Debt* In February 2005, the national debt was 7.634×10^{12} dollars and the U.S. population was 2.956×10^8 people (U.S. Treasury, www.treas.gov). What was each person's share of the debt at that time? $25,825

84. *Increasing Debt* In the three years prior to February of 2005, the national debt was increasing at an average rate of 1.44×10^9 dollars per day. If the debt keeps increasing at that rate, then what will be the amount of the debt in February of 2011? See the previous exercise. $\$1.079 \times 10^{13}$

85. *Energy from the Sun* Merely an average star, our sun is a swirling mass of dense gases powered by thermonuclear reactions. The great solar furnace transforms 5 million tons of mass into energy every second. How many tons of mass will be transformed into energy during the sun's 10 billion-year lifetime? 1.577×10^{24} tons

HINT Find the number of seconds in 10 billion years.

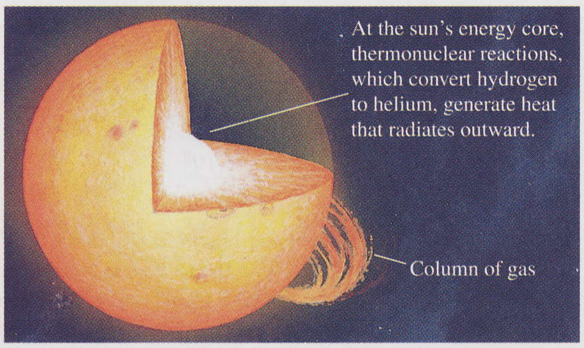


Figure for Exercise 85

86. *Orbit of the Earth* The earth orbits the sun in an approximately circular orbit with a radius of 1.495979×10^8 km. What is the area of the circle? 7.03×10^{16} km^2

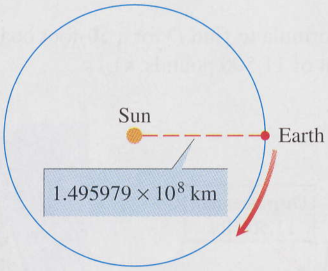


Figure for Exercises 86 and 87

87. *Radius of the Earth* The radius of the sun is 6.9599×10^5 km, which is 109.1 times the earth's radius. What is the radius of the earth? 6379 km

88. *Distance to the Sun* The distance from the sun to the earth is 1.495979×10^8 km. Use the fact that 1 km = 0.621 mi to find the distance in miles from the earth to the sun. 9.29×10^7 mi

89. *Mass of the Sun* The mass of the sun is 1.989×10^{30} kg and the mass of the earth is 5.976×10^{24} kg. How many times larger in mass is the sun than the earth? 3.3×10^5

90. *Speed of Light* If the speed of light is 3×10^8 m/sec, then how long does it take light from the sun to reach the earth? (See Exercise 88.) 8.3 min

HINT $T = D/R$

For Writing/Discussion

91. The integral powers of 10 are important in the decimal number system.

a. Use a calculator to evaluate 5365.4

$$5 \cdot 10^3 + 3 \cdot 10^2 + 6 \cdot 10^1 + 5 \cdot 10^0 + 4 \cdot 10^{-1}.$$

b. Without using a calculator, evaluate 9702.38

$$9 \cdot 10^3 + 7 \cdot 10^2 + 2 \cdot 10^0 + 3 \cdot 10^{-1} + 8 \cdot 10^{-2}.$$

c. Write a brief explanation of how you did part (b).

d. Write 9063.241 as a sum of integral multiples of powers of 10. Is your answer unique? Write a detailed description of how you did it. $9 \times 10^3 + 6 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1} + 4 \times 10^{-2} + 1 \times 10^{-3}$, not unique

e. Try to write 43,002.19 as a sum of integral multiples of powers of 10 using the description that you wrote for part (d). $4 \times 10^4 + 3 \times 10^3 + 2 \times 10^0 + 1 \times 10^{-1} + 9 \times 10^{-2}$

92. Many calculators can handle scientific notation for powers of 10 between −99 and 99. How can you be expected to compute $(4 \times 10^{220})^2$? Use the rules of exponents to get 1.6×10^{441}.

Thinking Outside the Box II & III

Powers of Three What is x if

$$\frac{1}{27} \cdot 3^{100} \cdot \frac{1}{81} \cdot 9^x = \frac{1}{3} \cdot 3^x? \quad -94$$

Common Remainders The numbers

1,576,231, 4,080,602, and 2,690,422

all yield the same remainder when divided by a certain natural number that is greater than one. What is the divisor and the remainder? Divisor 89, remainder 41