Systems of Linear Equations

I. Introduction –

1. General Form:

$$\mathbf{a_1}x + \mathbf{b_1}y = \mathbf{c_1}$$
$$\mathbf{a_2}x + \mathbf{b_2}y = \mathbf{c_2}$$

   where $\mathbf{a_i}$, $\mathbf{b_i}$, *and* $\mathbf{c_i}$ are (real no.) **constants**

2. Example:

$$2x + y = 4$$
$$x - y = 5$$

   $a_1 =$ ___ $b_1 =$ ___ $c_1 =$ ___

   $a_2 =$ ___ $b_2 =$ ___ $c_2 =$ ___

II. Solutions –

1. Definition: A "solution" is an ordered pair of numbers (x,y) which make both of the equations in the system true.
2. $(3,-2)$ is a solution to the system

$$2x + y = 4$$
$$x - y = 5$$

   since $\mathbf{2 \cdot 3 + (-2) = 4}$ *and* $\mathbf{3 - (-2) = 5}$

3. Example (p.273) / Problem #2

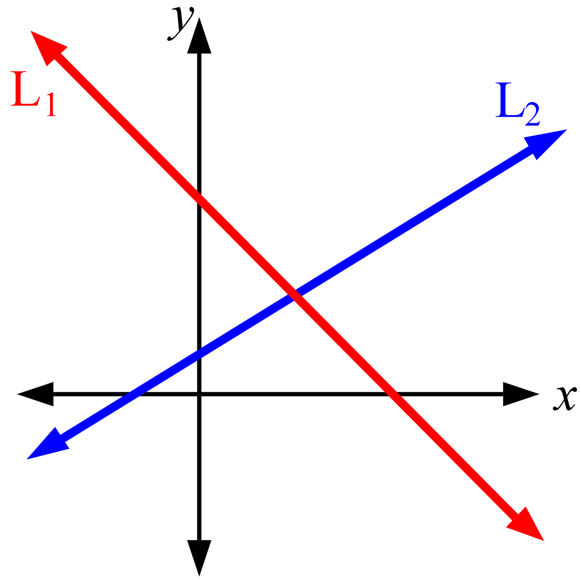
III. Methods for Solving –

1. Graphing (4.1)
2. Elimination (4.2), *a.k.a.* the addition method
3. Substitution (4.3)

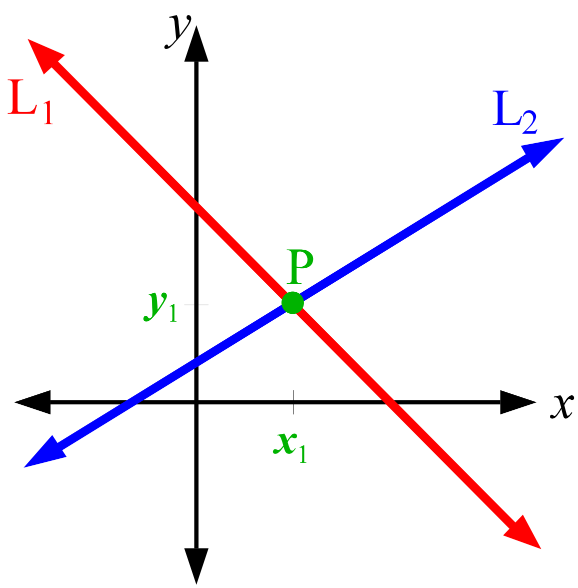
I. Graph the Equations...

L_1: $a_1 x + b_1 y = c_1$

L_2: $a_2 x + b_2 y = c_2$



II. Point of Intersection:



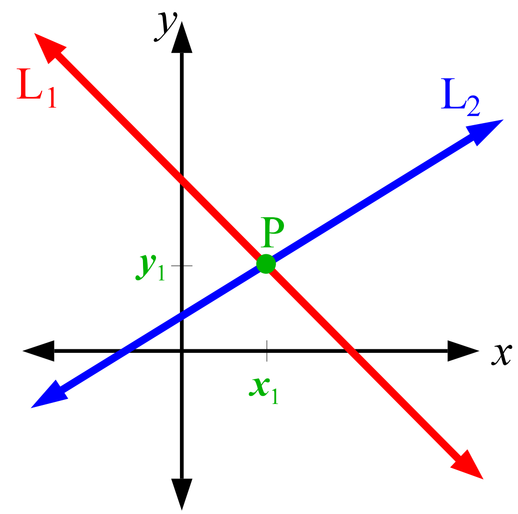
The point $\mathbf{P}(x_1, y_1)$ lies on both lines, therefore (x_1, y_1) is a solution to both equations; *i.e.*, the 2 numbers x_1 & y_1 are considered to be the "solution" to the system of equations...

III. Examples (p.273): Problems #6,22,**24**,28

IV. The Three (3) Possibilities:

1. A Unique Solution...

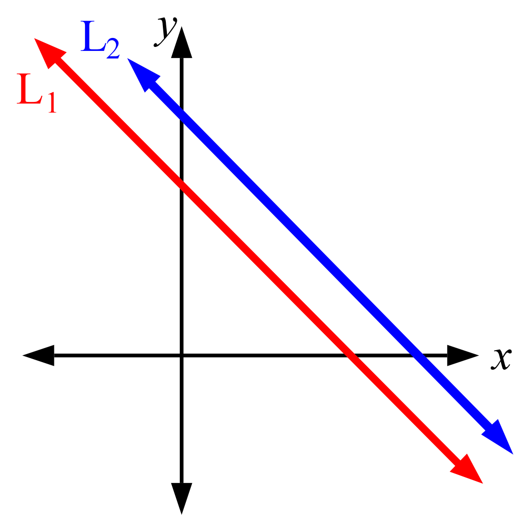
Two equations represent two distinct lines intersecting at the point P(x_1,y_1).



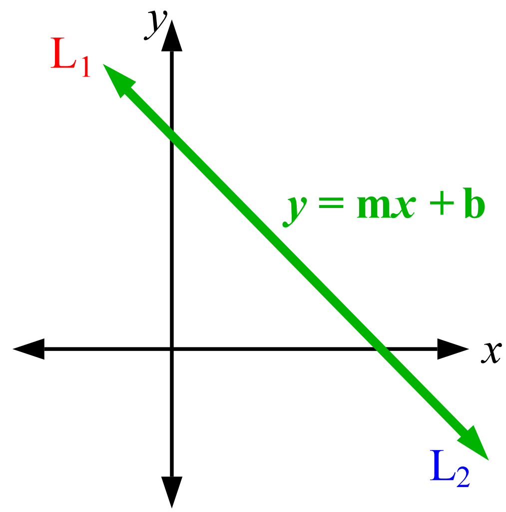
2. No Solution...

Two equations represent two parallel lines... (*i.e.*, there is no point of intersection)

"Inconsistent" system



IV. 3. Infinite # of Solutions...



Two equations represent only one line (note: every point on the satisfies both equations)

"Dependent" system

whose solutions are of the form...

$(x, mx+b)$ where x is <u>any</u> real #

V. Examples (p.273): Problems #**30**,32

HW: pp.273-276 / Problems #1,5,11,17,25,27,29, 31,43,45

Read pp.277-283 (section 4.2)