Systems of Linear Equations

I. Introduction –

1. General Form:

   \[ a_1x + b_1y = c_1 \]
   \[ a_2x + b_2y = c_2 \]

   where \( a_i, b_i, \) and \( c_i \) are (real no.) constants

2. Example:

   \[ 2x + y = 4 \]
   \[ x - y = 5 \]

III. Methods for Solving –

1. Graphing (4.1)
2. Elimination (4.2), a.k.a. the addition method
3. Substitution (4.3)
I. Graph the Equations...

Let \( L_1: a_1x + b_1y = c_1 \)
\( L_2: a_2x + b_2y = c_2 \)

II. Point of Intersection:

The point \( P(x_1, y_1) \) lies on both lines, therefore \( (x_1, y_1) \) is a solution to both equations; \( i.e. \), the 2 numbers \( x_1 \) & \( y_1 \) are considered to be the “solution” to the system of equations...

III. Examples (p.273): Problems #6, 22, 24, 28
IV. The Three (3) Possibilities:

1. A Unique Solution...

Two equations represent two distinct lines intersecting at the point \( P(x_1, y_1) \).

2. No Solution...

Two equations represent two parallel lines... (\( i.e. \), there is no point of intersection)

“Inconsistent” system
IV. 3. Infinite # of Solutions...

Two equations represent only one line (note: every point on the satisfies both equations)

“Dependent” system

whose solutions are of the form...

\((x, mx + b)\) where \(x\) is any real #

V. Examples (p.273): Problems #30,32

HW: pp.273-276 / Problems #1,5,11,17,25,27,29, 31,43,45

Read pp.277-283 (section 4.2)